Eur. Phys. J. C 56, 97–103 (2008)
DOI 10.1140/epjc/s10052-008-0630-6

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

# Neutrino decay as a possible interpretation of the MiniBooNE observation with unparticle scenario

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Received: 18 March 2008 /
Published online: 4 June 2008 – 

**Abstract.** In a new measurement on neutrino oscillations, $\nu_\mu \to \nu_e$, the MiniBooNE Collaboration observes an excess of electron-like events at low energy and the phenomenon may demand an explanation that obviously is beyond the oscillation picture. We propose that the heavier neutrino, $\nu_2$, decaying into a lighter one, $\nu_1$, via the transition process $\nu_\mu \to \nu_e + X$, where $X$ denotes any light products, could be a natural mechanism. The theoretical model we employ here is the unparticle scenario established by Georgi. We have studied two particular modes $\nu_\mu \to \nu_e + \mathcal{U}$ and $\nu_\mu \to \nu_e + \bar{\nu}_e + \nu_e$. Unfortunately, the number coming out of the computation is too small to explain the observation. Moreover, our results are consistent with the cosmology constraint on the neutrino life time and the theoretical estimation made by other groups; therefore, we can conclude that even though neutrino decay seems plausible in this case, it indeed cannot be the source of the peak at lower energy observed by the MiniBooNE Collaboration and there should be other mechanisms responsible for the phenomenon.

## 1 Introduction

Recently, the MiniBooNE Collaboration reported its results of searching for $\nu_\mu \to \nu_e$ oscillations [1]. In the experiment, the $\nu_\mu$ energy spectrum has a peak centered at 700 MeV and extends to 3000 MeV. For the oscillation range 475 MeV $< E_\nu <$ 1250 MeV, where $E_\nu$ is the energy of the produced neutrino, no significant excess of events is found. This result excludes the sizable appearance of $\nu_e$ via two neutrino oscillations and disfavors the previous LSND measurement [2, 3]. However, they observed that outside the oscillation range, there is a clear peak of electron-neutrino-like events ($96 \pm 17 \pm 20$ events) lying above background at 300 MeV $< E_\nu <$ 475 MeV. Although the origin of the excess is still under investigation, we may assume that these are indeed electron-neutrinos, at present. The beam is completely composed of $\nu_\mu$ and the oscillation can only produce $\nu_e$ with the same energy; therefore, the observation would be a serious challenge to the present theories. A reasonable explanation of the appearance of the low energy $\nu_e$ is needed. To answer this question, there are some interesting proposals, for example, in [4] the authors suggest a $(3+2)$ neutrino oscillation scenario where two sterile neutrinos are introduced to explain the MiniBooNE results, and Bodek [5] considered the internal bremsstrahlung as an alternative source of the excess $\nu_e$ events. Instead, in this work, we are looking for possible mechanisms other than the neutrino oscillation, supposing that there are only standard model (SM) neutrinos. An explanation that $\nu_\mu$ may decay into $\nu_e + X$, where $X$ denotes some possible light products, seems reasonable. Definitely, neutrino decay must be realized via interactions beyond the SM. The possible candidates of $X$ could be $\nu_e + \bar{\nu}_e$, light bosons (for example, axions etc.) and the unparticle, which we are going to explore in this work.

In fact, the idea of neutrino decay is not new. It has been put forward by several authors [6, 7]. The basic idea is to introduce a heavy, unstable neutrino (usually assuming a sterile one), which decays into a light neutrino or antineutrino plus a scalar particle. The interactions between the scalar particle and neutrinos are described by a lepton flavor violating effective lagrangian, which depends on the details of various new physics models. Instead, we suggest an alternative scenario, namely one in which the heavier $\nu_2$, which is a mass eigenstate and a component of the flavor eigenstate $\nu_\mu$, is the constituent of the beam and decays into a light neutrino $\nu_1$ and a scale-invariant unparticle proposed recently by Georgi [8].

It is well known that at a very high energy scale, the unparticle physics contains the SM fields and a sector of the Banks–Zaks field (defined in [9]) with a non-trivial infrared fixed point. Below an energy scale $\Lambda_{\mathcal{U}}$, which is of order of TeV, the Banks–Zaks fields are matched onto a scale-invariant unparticle sector. The unparticle is different from ordinary particles, as it has no mass, since the mass term breaks scale invariance, but the Lorentz-invariant four-momentum square needs not be zero, $P^2 \geq 0$. The scale dimension of the unparticle is in general fractional, rather

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than an integral number (the dimension for a fermion is half-integer). This special characteristic brings us a natural explanation of the shape of the low energy $\nu_e$ bump observed by the MiniBooNE Collaboration. If $\nu_2$ decays into a $\nu_1$ and a real scalar particle, where the $\nu_i$ are neutrino mass eigenstates, it is a two-body decay in which the energy spectrum of the produced $\nu_e$ should be discrete. It is contrary to the observation where the energy spectrum of the produced $\nu_e$ is continuous. Indeed, the incident $\nu_\mu$ beam has an energy distribution that may result in a natural energy spread for the produced electron-neutrino; however, it demands that the shape of the $\nu_e$ spectrum must be similar to that of the incident $\nu_\mu$ beam. Instead, if the produced $X$ is an unparticle, the energy spectrum of $\nu_e$ would naturally spread, and it may be more consistent with the present measurements. The interactions between the unparticle and the SM particles are described in the framework of low energy effective theory and lead to various interesting features in the phenomenology. There have been many phenomenological explorations on possible observable effects caused by unparticles [8, 10–41] and much more are coming up.

The MiniBooNE results indicate that the energy of the events of excess is about half of the peak position at the energy spectrum of the muon-neutrino. As discussed above, we suggest a decay mode $\nu_2 \to \nu_1 + \mathcal{U}$, where $\mathcal{U}$ denotes the unparticle and a consequent transition $\nu_\mu \to \nu_e + \mathcal{U}$ might be observed, namely $\nu_\mu$ and $\nu_e$ are not physical eigenstates but are eigenstates of the weak interaction and can be caught by a detector as the appearance of $\nu_e$ at lower energy. As indicated in [8], the unparticle stuff with scale dimension $d_\mathcal{U}$ cannot be "seen" directly; it would manifest itself as a missing energy. When the scale dimension $d_\mathcal{U}$ is not very large, the energy spectrum of the electron-neutrino may fall into the allowed range of the MiniBooNE measurements. This process has also been considered in [27]. A transition into a three-body final state $\nu_\mu \to \nu_e + \nu_e + \bar{\nu}_e$ is also a possible process to explain the MiniBooNE data. The two decay modes $\nu_\mu \to \nu_e + \mathcal{U}$ and $\nu_\mu \to \nu_e + \nu_e + \bar{\nu}_e$ are both lepton flavor violating processes and can only occur via a new physics mechanism beyond the SM.

If the mechanism proposed above can explain the observed peak, which depends on its decay width, the possible detection rate of the number of $\nu_e$ is roughly

$$N_{\nu_e} \sim N_0 (1 - \mathrm{e}^{-t/\tau}) \eta \,, \quad (1)$$

where $N_0$ is the muon-neutrino number, $\tau$ is the life time, which should be calculated in the aforementioned scenario, and $\eta$ is a detection rate, which is also very small, say, $10^{-10}$ or even smaller. $t$ is the time of flight from the source to the detector, and since the speed of the beam neutrino is very close to the speed of light, $t \sim L/c$, where $L$ is the distance, approximately 500 m in the MiniBooNE experiments. In addition, the ratio would be further suppressed by the time dilation factor $\gamma = m/E$. Since $L$ is only several hundred meters, to make a sizable ratio that can be observed, $\tau$ must not be large. We will obtain its value by taking as input all the concerned model parameters, which are fixed by fitting data of other experiments into our numerical computation.

As is well known, neutrino oscillations have been observed in solar, atmospheric, accelerator neutrino experiments and the present theoretical studies almost completely confirm the MSW mechanism. The relevant mixing parameters and the mass square differences are determined by fitting the data, even though the absolute values of the neutrino masses are still not fixed yet. Theoretically determining the parameters, possible neutrino decays are not taken into account seriously. How to reconcile the neutrino decay with the present theoretical works on the neutrino oscillation is an open question; one should explore if there exists a discrepancy between the theoretical predictions and the data. Obviously if the decay rate is sufficiently small, one does not need to modify the present theoretical framework of neutrino oscillations, but if the decay rate is not too small, the data fitting should be re-considered, and therefore the MiniBooNE result indeed provides a new challenge to neutrino physics and we will return to this topic in [42].

## 2 Neutrino decays in unparticle physics

We start with a brief review of unparticle physics. First, let us consider an unparticle $\mathcal{U}$ with scale dimension $d_\mathcal{U}$ and momentum $P$. The unparticle momentum satisfies the constraint $P^2 \geq 0$. According to [19], the unparticle stuff can be viewed as a tower of massive particles with mass spacing tending to zero. Scale invariance provides the most important constraint on the properties of unparticles. The two-point function of the scalar unparticle field operator $O_\mathcal{U}$ is written as

$$\langle 0|O_\mathcal{U}(x)O^\dagger_\mathcal{U}(0)|0\rangle = \int \frac{\mathrm{d}^4P}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}P\cdot x} |\langle 0|O_\mathcal{U}(0)|P\rangle|^2 \rho(P^2) \,, \quad (2)$$

where $|P\rangle$ is the unparticle state with momentum $P$ and the phase space factor is

$$|\langle 0|O_\mathcal{U}(0)|P\rangle|^2 \rho(P^2) = A_{d_\mathcal{U}} \theta(P^0)\theta(P^2)(P^2)^{d_\mathcal{U}-2} \,, \quad (3)$$

where

$$A_{d_\mathcal{U}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_\mathcal{U}}} \frac{\Gamma(d_\mathcal{U}+1/2)}{\Gamma(d_\mathcal{U}-1)\Gamma(2d_\mathcal{U})} \,. \quad (4)$$

For the vector unparticle field $O^\mu_\mathcal{U}$, we have

$$\begin{aligned}\langle 0|O^\mu_\mathcal{U}(0)|P\rangle\langle P|O^\nu_\mathcal{U}(0)|0\rangle\rho(P^2) \\ = A_{d_\mathcal{U}} \theta(P^0)\theta(P^2)(-g^{\mu\nu} + P^\mu P^\nu/P^2)(P^2)^{d_\mathcal{U}-2} \,,\end{aligned} \quad (5)$$

where the transverse condition $\partial_\mu O^\mu_\mathcal{U} = 0$ is required. The Lorentz structure of an unparticle may also be tensor [11, 33] or spinor [12]. In this study, we restrict our discussion to scalar and vector unparticles. Obviously a similar analysis can be done for tensor and spinor unparticles.

As regards the virtual effects, the propagator of the scalar unparticle field is given by

$$\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}P\cdot x}\langle 0|TO_{\mathcal{U}}(x)O_{\mathcal{U}}(0)|0\rangle = \mathrm{i}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{1}{(P^2+\mathrm{i}\epsilon)^{2-d_{\mathcal{U}}}}\mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}\,, \quad (6)$$

and for the vector unparticle field, the propagator is

$$\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}P\cdot x}\langle 0|TO^{\mu}_{\mathcal{U}}(x)O^{\nu}_{\mathcal{U}}(0)|0\rangle = \mathrm{i}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{-g^{\mu\nu}+P^{\mu}P^{\nu}/P^2}{(P^2+\mathrm{i}\epsilon)^{2-d_{\mathcal{U}}}}\mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}\,. \quad (7)$$

The function $\sin(d_{\mathcal{U}}\pi)$ in the denominator implies that the scale dimension $d_{\mathcal{U}}$ cannot be integer for $d_{\mathcal{U}} > 1$, in order to avoid a singularity. The phase factor $\mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}$ provides a $CP$ conserving phase, which produces peculiar interference effects in high energy scattering processes [10, 11, 33] and $CP$ violation in $B$ decays [13, 29].

In this study, we will discuss interactions between the unparticles and neutrinos. The framework that describes these interactions is a low energy effective theory. For our purposes, the coupling of an unparticle to neutrinos ($\nu_\mu$ and $\nu_e$) is given in the form

$$\mathcal{L}_{\mathrm{eff}} = \frac{c_S^{\nu_\alpha\nu_\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{\nu}_\beta\gamma_\mu(1-\gamma_5)\nu_\alpha\partial^\mu O_{\mathcal{U}} + \frac{c_V^{\nu_\alpha\nu_\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\bar{\nu}_\beta\gamma_\mu(1-\gamma_5)\nu_\alpha O^\mu_{\mathcal{U}} + \mathrm{h.c.} \quad (8)$$

Here, we have used the $V-A$ type current as in the SM. The $c_S$ and $c_V$ are dimensionless coefficients. The $\nu_\alpha$ and $\nu_\beta$ are weak eigenstates with different flavor numbers $\alpha$ and $\beta$.

As in [27], the neutrino decay is conveniently represented in the basis of mass eigenstates $\nu_i$ ($i = 1, 2$; we only consider two generations in this case). The interactions between unparticle and neutrinos are rewritten by

$$\frac{c_S^{\nu_i\nu_j}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{\nu}_j\gamma_\mu(1-\gamma_5)\nu_i\partial^\mu O_{\mathcal{U}} + \frac{c_V^{\nu_i\nu_j}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\bar{\nu}_j\gamma_\mu(1-\gamma_5)\nu_i O^\mu_{\mathcal{U}} + \mathrm{h.c.} \quad (9)$$

The relation between the coupling coefficients $c_{S(V)}^{\nu_\alpha\nu_\beta}$ and $c_{S(V)}^{\nu_i\nu_j}$ can be obtained from the neutrino mixing matrix. For the simple case considering two neutrino mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}. \quad (10)$$

The coefficients in the mass basis are related to those in the flavor basis by

$$c_{S(V)}^{\nu_1\nu_2} = \cos^2\theta c_{S(V)}^{\nu_\alpha\nu_\beta}\,. \quad (11)$$

For a maximal mixing, where $\theta = \pi/4$, $c_{S(V)}^{\nu_1\nu_2} = \frac{1}{2}c_{S(V)}^{\nu_\alpha\nu_\beta}$. The coefficients in the different bases differ by a constant factor.

### 2.1 The decay of $\nu_2 \to \nu_1 + \mathcal{U}$

Assuming two generation neutrinos and that the heavier one is $\nu_2$ and the lighter one is $\nu_1$, the decay of $\nu_\mu \to \nu_e + \mathcal{U}$ is realized via the transition $\nu_2 \to \nu_1 + \mathcal{U}$, which is a typical lepton flavor violating process and its Feynman diagram is depicted in Fig. 1. Here, it is natural to assume that the basis of the interaction between unparticle and neutrino is the same as the weak interaction, namely, $\nu_e$ and $\nu_\mu$ are the eigenstates of the interaction. The final unparticle is invisible and behaves as a missing energy. The decay $\nu_2 \to \nu_1 + \mathcal{U}$ seems to be a two-body process. But it is different from the common case with two final particles whose momenta are single-valued and fixed. For the unparticle case, the energy of $\nu_1$ depends on the momentum square of the unparticle, $P^2$, which only is constrained by the condition $P^2 \geq 0$. Namely, one may expect that $P^2$ can vary within a range $0 \leq P^2 \leq P^2_{\mathrm{max}}$, where $P_{\mathrm{max}}$ would be determined by the momenutum conservation in $\nu_\mu$ decay. Thus, the varying $P^2$ causes a continuous energy spectrum of $E_{\nu_e}$ and this is a characteristic effect of the unparticle.

In order to make the process realizable, the mass of $\nu_2$ should be larger than that of $\nu_1$, which is the called the normal order in the literature. Without losing generality, we further assume $m_{\nu_2} \gg m_{\nu_1}$ and neglect $m_{\nu_1}$ in the analysis below. The differential decay rate of $\nu_2(p_2) \to \nu_1(p_1) + \mathcal{U}(q)$ is

$$\mathrm{d}\Gamma = \frac{1}{2E_{\nu_2}}\frac{1}{2}\sum_{\mathrm{spins}}|\mathcal{M}|^2\mathrm{d}\Phi(p)\,, \quad (12)$$

where the phase space factor $\mathrm{d}\Phi(p)$ is

$$\mathrm{d}\Phi(p) = (2\pi)^4\delta^4(p_2-p_1-q)\left[2\pi\theta\left(p_1^0\right)\delta\left(p_1^2\right)\frac{\mathrm{d}^4p_1}{(2\pi)^4}\right] \times\left[A_{d_{\mathcal{U}}}\theta(q^0)\theta(q^2)(q^2)^{d_{\mathcal{U}}-2}\frac{\mathrm{d}^4q}{(2\pi)^4}\right], \quad (13)$$

with $q = p_2 - p_1$, and the Lorentz-invariant amplitude square is

$$\frac{1}{2}\sum_{\mathrm{spins}}|\mathcal{M}|^2 = \frac{\left|c_S^{\nu_1\nu_2}\right|^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}4p_2^2(p_1\cdot p_2) \quad (14)$$

for the scalar unparticle and

$$\frac{1}{2}\sum_{\mathrm{spins}}|\mathcal{M}|^2 = \frac{\left|c_V^{\nu_1\nu_2}\right|^2}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}-2}}4\left[2(p_1\cdot p_2)+\frac{p_2^2(p_1\cdot p_2)}{q^2}\right], \quad (15)$$

for the vector unparticle.

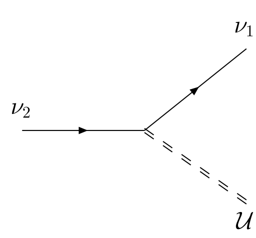


**Fig. 1.** The diagram for the decay of $\nu_2 \to \nu_1 + \mathcal{U}$. The *double dashed lines* represent the unparticle

In the rest frame of $\nu_2$, it is straightforward to derive the differential decay rate over the $\nu_1$ energy $E_1$ and the decay rate of $\nu_2(p_2) \to \nu_1(p_1) + \mathcal{U}(q)$:

$$\frac{\mathrm{d}\Gamma_S}{\mathrm{d}E_1} = \frac{\left|c_S^{\nu_1\nu_2}\right|^2 A_{d_\mathcal{U}}}{2\pi^2 \Lambda_\mathcal{U}^{2d_\mathcal{U}}} \frac{m_{\nu_2}^2 E_1^2}{\left(m_{\nu_2}^2 - 2m_{\nu_2}E_1\right)^{2-d_\mathcal{U}}} \theta(m_{\nu_2} - 2E_1)\,,$$

$$\Gamma_S = \frac{\left|c_S^{\nu_2\nu_1}\right|^2 A_{d_\mathcal{U}}}{8\pi^2 d_\mathcal{U}(d_\mathcal{U}^2 - 1)} m_{\nu_2} \left(\frac{m_{\nu_2}}{\Lambda_\mathcal{U}}\right)^{2d_\mathcal{U}}, \tag{16}$$

$$\frac{\mathrm{d}\Gamma_V}{\mathrm{d}E_1} = \frac{\left|c_V^{\nu_1\nu_2}\right|^2 A_{d_\mathcal{U}}}{2\pi^2 \Lambda_\mathcal{U}^{2d_\mathcal{U}-2}} \frac{m_{\nu_2}^2 E_1^2 \left[1 + 2\left(1 - 2\frac{E_1}{m_{\nu_2}}\right)\right]}{\left(m_{\nu_2}^2 - 2m_{\nu_2}E_1\right)^{3-d_\mathcal{U}}} \times \theta(m_{\nu_2} - 2E_1)\,,$$

$$\Gamma_V = \frac{3\left|c_V^{\nu_1\nu_2}\right|^2 A_{d_\mathcal{U}}}{8\pi^2 d_\mathcal{U}(d_\mathcal{U}+1)(d_\mathcal{U}-2)} m_{\nu_2} \left(\frac{m_{\nu_2}}{\Lambda_\mathcal{U}}\right)^{2d_\mathcal{U}-2}, \tag{17}$$

with $d_\mathcal{U} > 1$ for the scalar unparticle and $d_\mathcal{U} > 2$ for the vector unparticle.

In this study, we are concerned with the observation in the laboratory frame where the initial muon neutrino moves nearly with the speed of light. The energy of $\nu_2$ is at the order of several hundred MeV and is much larger than its inertial mass, $E_2 \gg m_{\nu_2}$. The invariant amplitude square in the laboratory frame depends on the angle $\theta'$ between the moving directions of muon- and electron-neutrinos. Indeed, we need to do some treatments to get an analytical result, i.e. boost the result in the rest frame of $\nu_2$ into the laboratory frame. The momentum of $\nu_2$ is approximated by $|\mathbf{p}_2| = \sqrt{E_2^2 - m_{\nu_2}^2} \cong E_2\left(1 - \frac{m_{\nu_2}^2}{2E_2^2}\right)$ and the momentum product $p_2 \cdot p_1 \cong E_1 E_2 \left(\frac{m_{\nu_2}^2}{2E_2^2} + 1 - \cos\theta'\right)$. From $q^2 \geq 0$, the range of $\cos\theta'$ is determined to be $0 \leq 1 - \cos\theta' \leq \frac{m_{\nu_2}^2}{2E_2E_1}\left(1 - \frac{E_2}{E_1}\right)$, which means that the three-momentum of the produced electron-neutrino is almost parallel to that of the muon-neutrino. After performing an integration over $\cos\theta'$, we obtain the differential decay rate of $\nu_2(p_2) \to \nu_1(p_1) + \mathcal{U}(q)$ in the laboratory frame:

$$\frac{\mathrm{d}\Gamma_S}{\mathrm{d}E_1} = \frac{\left|c_S^{\nu_1\nu_2}\right|^2 A_{d_\mathcal{U}}}{4\pi^2} \left(\frac{m_{\nu_2}^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}} \times \frac{m_{\nu_2}^2}{E_\mu^2} \frac{[1 + y(d_\mathcal{U} - 1)](1-y)^{d_\mathcal{U}-1}}{d_\mathcal{U}(d_\mathcal{U}-1)} \theta(E_2 - E_1) \tag{18}$$

for the scalar unparticle with $y \equiv \frac{E_1}{E_2}$ and $d_\mathcal{U} > 1$. For the vector unparticle, the differential decay rate is

$$\begin{aligned}\frac{\mathrm{d}\Gamma_V}{\mathrm{d}E_1} = {} & \frac{\left|c_V^{\nu_1\nu_2}\right|^2 A_{d_\mathcal{U}}}{16\pi^2} \left(\frac{m_{\nu_2}^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-1} \frac{m_{\nu_2}^2}{E_2^2} (1-y)^{d_\mathcal{U}-2} \\ & \times \Gamma(d_\mathcal{U}+1) \Bigg[ y(3-2y) \frac{{}_2F_1(1,3;d_\mathcal{U}+2;1)}{\Gamma(d_\mathcal{U}+2)} \\ & + (3 - 7y + 4y^2) \frac{{}_2F_1(2,3;d_\mathcal{U}+3;1)}{\Gamma(d_\mathcal{U}+3)} \\ & - 4(1-y)^2 \frac{{}_2F_1(3,3;d_\mathcal{U}+4;1)}{\Gamma(d_\mathcal{U}+4)} \Bigg] \theta(E_2 - E_1)\,, \end{aligned} \tag{19}$$

with $d_\mathcal{U} > 2$ and where ${}_2F_1(a,b;c;z)$ is the hypergeometric function. The decay rates $\Gamma_S$ and $\Gamma_V$ can be obtained, and the final results differ from (16) and (17) by a Lorentz factor $\frac{m_{\nu_2}}{E_2}$.

### 2.2 The three-body decay of $\nu_2 \to \nu_1 + \bar{\nu}_1 + \nu_1$

As briefly discussed in the introduction, there is another possibility to observe a continuous energy spectrum of $\nu_1$. Now, we turn to the three-body decay of $\nu_2$ in the framework of the unparticle: $\nu_2 \to \nu_1 + \bar{\nu}_1 + \nu_1$. The Feynman diagram is depicted in Fig. 2, where the unparticle serves as an intermediate agent. Because the final states have two electron-neutrinos, there are two diagrams in the process and one needs to consider the anti-symmetrization of the two identical fermions. We consider only the vector unparticle part, because the scalar unparticle contribution is proportional to the light neutrino mass and should be very suppressed. According to the effective interaction of neutrinos and unparticle, the decay amplitude of $\nu_2(p_0) \to \nu_1(p_1) + \nu_1(p_2) + \bar{\nu}_1(p_3)$ is

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2\,,$$

$$\mathcal{M}_1 = -\frac{c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}}{\Lambda_\mathcal{U}^{2d_\mathcal{U}-2}} \frac{A_{d_\mathcal{U}} \mathrm{e}^{-\mathrm{i}\phi}}{2\sin d_\mathcal{U}\pi} \times \frac{\bar{u}(p_1)\gamma_\mu(1-\gamma_5)u(p_0)\bar{u}(p_2)\gamma_\mu(1-\gamma_5)v(p_3)}{\left(q_1^2\right)^{2-2d_\mathcal{U}}}\,,$$

$$\mathcal{M}_1 = +\frac{c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}}{\Lambda_\mathcal{U}^{2d_\mathcal{U}-2}} \frac{A_{d_\mathcal{U}} \mathrm{e}^{-\mathrm{i}\phi}}{2\sin d_\mathcal{U}\pi} \times \frac{\bar{u}(p_2)\gamma_\mu(1-\gamma_5)u(p_0)\bar{u}(p_1)\gamma_\mu(1-\gamma_5)v(p_3)}{\left(q_2^2\right)^{2-2d_\mathcal{U}}}\,, \tag{20}$$

where $\phi = (d_\mathcal{U} - 2)\pi$, $q_1 = p_0 - p_1$ and $q_2 = p_0 - p_2$. The amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ represent contributions from Fig. 2a and b, respectively. In the derivations, we have neglected $q_1^\mu q_1^\nu / q_1^2$ and $q_2^\mu q_2^\nu / q_2^2$ terms. The square of the invariant matrix element is

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{\left|c_V^{\nu_1\nu_2} c_V^{\nu_1\nu_1}\right|^2}{8\Lambda_\mathcal{U}^{4d_\mathcal{U}-4}} \frac{A_{d_\mathcal{U}}^2}{4(\sin d_\mathcal{U}\pi)^2} \times \left[\frac{1}{\left(q_1^2\right)^{2-2d_\mathcal{U}}} + \frac{1}{\left(q_2^2\right)^{2-2d_\mathcal{U}}}\right]^2 256(p_0 \cdot p_3)(p_1 \cdot p_2)\,. \tag{21}$$

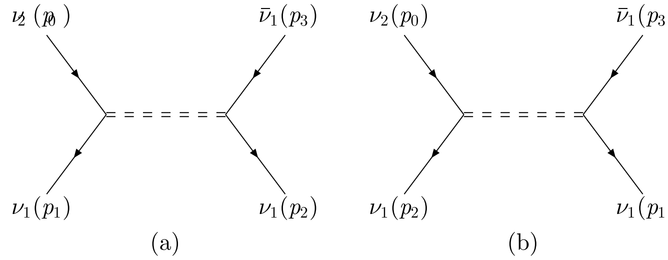


**Fig. 2.** The Feynman diagram for the three-body decay of $\nu_2 \to \nu_1 + \nu_1 + \bar{\nu}_1$

For the three-body decays, we work in the rest frame of the muon-neutrino. As will be shown later, the three-body decay rate of $\nu_2 \to \nu_1 + \bar{\nu}_1 + \nu_1$ is much smaller than that of the two-body decay $\nu_2 \to \nu_1 + \mathcal{U}$. This conclusion does not depend on which reference frame we choose. In the rest frame of $\nu_2$, the three-body kinematics are described in terms of the final neutrino energies, $E_1$ and $E_2$, by

$$p_0 \cdot p_3 = m_{\nu_\mu}(m_{\nu_\mu} - E_1 - E_2)\,,$$
$$p_1 \cdot p_2 = m_{\nu_\mu}\left(E_1 + E_2 - \frac{m_{\nu_\mu}}{2}\right),$$
$$q_1^2 = (p_0 - p_1)^2 = m_{\nu_\mu}^2 - 2m_{\nu_\mu}E_1\,,$$
$$q_2^2 = (p_0 - p_2)^2 = m_{\nu_\mu}^2 - 2m_{\nu_\mu}E_2\,. \quad (22)$$

Thus, the differential decay rate is

$$\mathrm{d}\Gamma_V(\nu_2 \to \nu_1 + \nu_1 + \bar{\nu}_1) = \frac{1}{(2\pi)^3}\frac{1}{8m_{\nu_2}}\frac{1}{2}\frac{1}{2}|\mathcal{M}|^2\,\mathrm{d}E_1\,\mathrm{d}E_2\,, \quad (23)$$

where the integration range is $0 \le E_1 \le \frac{m_{\nu_2}}{2}$ and $\frac{m_{\nu_2}}{2} - E_1 \le E_2 \le \frac{m_{\nu_2}}{2}$.

Note that the similar lepton flavor violating processes $\mu^- \to e^- + \mathcal{U}$, $\mu^- \to e^- + e^- + e^+$ have been studied in [16, 23], and their formulations are quite similar to ours.

## 3 Analysis of the concerned phenomenology

For the neutrino accelerator experiment, the neutrinos fly over a baseline over a distance $L$ before reaching the final detectors. If the neutrino decays as we suggested, the number of final electron-neutrinos produced from muon-neutrino decay is

$$N_{\nu_e} = N_0 \exp\left[1 - \left(-\frac{t}{\tau_{\mathrm{lab}}}\right)\right] \approx N_0 \frac{L}{c\tau_\nu}\frac{m}{E}\,, \quad (24)$$

with $N_0$ the initial muon-neutrino number, $\tau_{\mathrm{lab}}$ and $\tau_\nu$ are the neutrino life times in the laboratory and the rest frame, respectively. In the MiniBooNE measurement, $L/E \sim 500\,\mathrm{m}/500\,\mathrm{MeV}$ and $N_{\nu_e} \sim 100$. The ratio of neutrino life time over mass $\tau_\nu/m_\nu \sim \frac{N_0}{N_{\nu_e}}10^{-14}\,\mathrm{s/eV}$. When $N_0/N_{\nu_e} \sim 10^{10}$, $\tau_\nu/m_\nu \sim 10^{-4}$, and when $N_0/N_{\nu_e} \sim 10^5$, $\tau_\nu/m_\nu \sim 10^{-9}$.

At present, our knowledge of the neutrino mass is mainly obtained from the neutrino oscillation data. The squared mass difference of the mass eigenstates, $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, is observed to be [43] $\Delta m^2 \sim 8 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2 \sim 3 \times 10^{-3}\,\mathrm{eV}^2$. We will not use the LSND result ($\Delta m^2 \sim 1\,\mathrm{eV}^2$), since it is disfavored by other neutrino experiments and the new MiniBooNE measurements. From these data, we choose $m_2 = 50\,\mathrm{meV}$ as the upper limit in our numerical calculations.

Firstly, we make the observation that the rate of three-body decay $\nu_\mu \to \nu_e + \bar{\nu}_e + \nu_e$ is much smaller than the one of the two-body process $\nu_\mu \to \nu_e + \mathcal{U}$. As an illustration, we choose the parameters as $c_S = c_V = 1$, $\Lambda_{\mathcal{U}} = 1\,\mathrm{TeV}$, $d_{\mathcal{U}} = 1.1$ for scalar and $d_{\mathcal{U}} = 2.1$ for vector unparticles. The parameters satisfy the cosmological constraints. For the vector unparticle contribution, the rates of two-body and three-body decays are

$$\Gamma_V(\nu_\mu \to \nu_e + \mathcal{U}) = 4 \times 10^{-34}\,\mathrm{eV}\,,$$
$$\Gamma_V(\nu_\mu \to \nu_e + \bar{\nu}_e + \nu_e) = 1 \times 10^{-66}\,\mathrm{eV}\,. \quad (25)$$

The three-body decay rate is more than 30 orders smaller than that of the two-body case. The tiny ratio is due to the very weak coupling between unparticle and neutrinos (and there are two such vertices for the process; see Fig. 2) and the small neutrino mass. This observation is analogous to the process of $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$, where the decay rate is proportional to $G_\mathrm{F}^2$ and $m_\mu^5$. If we only use the three-body decay, the life time of a neutrino is so long that the muon-neutrino will never decay and the bump observed in the experiment cannot be explained by neutrino decay at all. Thus, we can safely neglect the contributions from the three-body decays and approximate $\Gamma_\nu = \Gamma(\nu_\mu \to \nu_e + \mathcal{U})$.

Secondly, we discuss the constraints of the unparticle parameters from neutrino decays. As discussed above, the MiniBooNE experiments put a bound for the neutrino life time in the rest frame $\tau_\nu/m_\nu \sim 10^{-4}\,\mathrm{s/eV}$. We take this bound for our analysis and discuss three possibilities. For the illustration, we are restricted to the case of the vector unparticle. (1) We fix $d_{\mathcal{U}} = 2.1$, $\Lambda_{\mathcal{U}} = 1\,\mathrm{TeV}$, and constrain $c_V$ by $c_V > 10^{11}$. The coupling constants are found to be much larger than the order 1. (2) We fix $c_S = c_V = 1$, $\Lambda_{\mathcal{U}} = 1\,\mathrm{TeV}$, and constrain $0 < d_{\mathcal{U}} - 2 < 10^{-7}$. The scale dimension will be nearly equal to 2. (3) We fix $c_S = c_V = 1$, $d_{\mathcal{U}} = 2.1$, and constrain $\Lambda_{\mathcal{U}} < 10^{-9}\,\mathrm{TeV}$, which is obviously impossible. Thus, if the neutrino decays as suggested, the unparticle parameters have to fall into a very unnatural space. On the other hand, if the unparticle parameters are chosen in a reasonable space, the neutrino life time will be so long that they will not decay when flying over the distance $L \sim 500\,\mathrm{m}$ in the MiniBooNE experiment.

Thirdly, we discuss the relative energy spectrum of the final electron-neutrino. Figure 3 plots the initial $\nu_\mu$ energy spectrum. The distribution is a quasi-Gaussian function, which peaks around 700 MeV. If the muon-neutrino decays to an electron-neutrino and a conventional particle, the energy spectrum of the electron-neutrino has the same distribution as that the muon-neutrino beam. From the data of excess events plotted in Fig. 4, obviously the $\nu_e$ energy spectrum is not consistent with the data, no matter for low energy or high energy. The experimental data show that the excess of $\nu_e$ events is a decreasing function rather than a Gaussian distribution in the energy range $0.3\,\mathrm{GeV} < E_{\nu_e} < 1.0\,\mathrm{GeV}$. This excludes the neutrino decays with conventional particles. The unparticle is not a regular particle and has no a fixed mass. The energy distribution of $\nu_e$ with an unparticle being in the final state is different from the initial $\nu_\mu$ spectrum, and the final energy distribution of $\nu_e$ depends on both effects. Combining the $\nu_\mu$ energy spectrum (energy spreading of the muon-neutrino beam) and the differential ratios of the neutrino decay given in (18) and (19), the final electron-neutrino energy distribution is depicted in Figs. 4 and 5 for scalar and vector unparti-

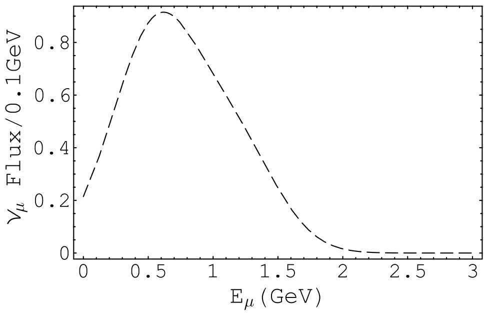


**Fig. 3.** The energy spectrum for the $\nu_\mu$ beam in the laboratory frame

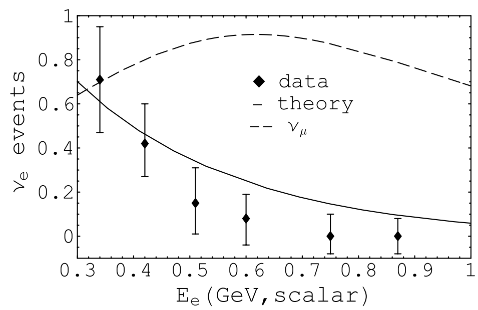


**Fig. 4.** The energy spectrum for the decay of $\nu_\mu \to \nu_e + \mathcal{U}$ with a scalar unparticle in the laboratory frame where $d_{\mathcal{U}} = 1.1$, $\Lambda_{\mathcal{U}} = 1$ TeV and $c_S = c_V = 1$

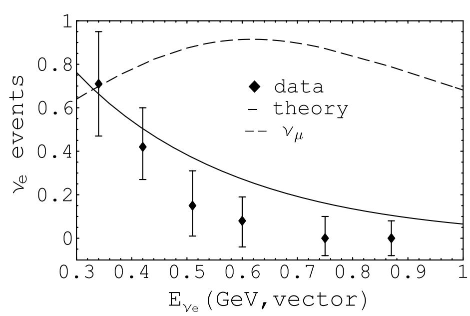


**Fig. 5.** The energy spectrum for the decay of $\nu_\mu \to \nu_e + \mathcal{U}$ with a vector unparticle where $d_{\mathcal{U}} = 2.05$, $\Lambda_{\mathcal{U}} = 1$ TeV and $c_S = c_V = 1$

cles, respectively. Within the non-oscillation region, i.e., at low energy, $0.3\,\text{GeV} < E_{\nu_e} < 0.45\,\text{GeV}$, the theoretical prediction is well consistent with the experimental data. For the energy range $0.5\,\text{GeV} < E_{\nu_e} < 1.05\,\text{GeV}$, the theoretical prediction is slightly larger than the data. It is noted that in the above figures only the relative size is estimated; the absolute magnitude is very small.

## 4 Discussion and conclusions

Motivated by the new measurement of the MiniBooNE Collaboration, which observed an excess of electron-like events at low energy, we started to search for a possible mechanism beyond the SM to explain this phenomenon, and the first idea to hit our mind was that $\nu_2$ might decay into $\nu_1$ accompanied by some other very light products. We should test if this scenario can produce results that are theoretically plausible and can explain the data.

There may be several possible modes; the first one is $\nu_2 \to \nu_1 + \bar{\nu}_1 + \nu_1$, which is a three-body decay, the second one is $\nu_2 \to \nu_1 + a$, where $a$ is a single boson particle, for example an axion etc., and the third one is $\nu_2 \to \nu_1 + \mathcal{U}$, where $\mathcal{U}$ represents an unparticle. All the three possibilities cannot be realized in the framework of the SM, so new physics beyond the SM is necessary. The first one was numerically estimated in this work and our results indicate that the decay rate determined by the three-body decay mode is too small and is ruled out immediately. The second mode is two-body decay; therefore the spectrum of the electron-neutrinos is discrete and it is not consistent with the measurement of the MiniBooNE. Even though we consider the energy spreading of the incident muon-neutrino beam, the shape of the resultant electron-neutrino bump cannot be well understood in this scenario. Therefore, the third candidate is the one most favorable. In this work, we work out the formulae of neutrino decays within the framework of unparticle physics. The formulae in the laboratory frame are given for the first time.

The smallness of the decay width given by our numerical results indicates that the unparticle scenario may not explain the excess of electron-neutrinos at low energy. The life time predicted in the unparticle model is qualitatively consistent with the cosmological constraint [44], which is about $10^{17}$ s. By (1), we know that the suppression of $\exp(-t/10^{17})$ with $t \sim 10^{-7}$ s would kill any possibility of observing a decay event. The reasons are (1) the very tiny neutrino mass, and (2) the very weak interactions between the unparticle and neutrino. Since our numerical results are consistent with the cosmology constraints and the results by other authors [27], we may be convinced that the calculation is right, but the proposal does not work here.

Thus, in this work, we definitely obtain a negative conclusion: that the peak of the electron-neutrino at lower energy observed by the MiniMoone Collaboration cannot be explained by the neutrino decay. But the phenomenon is there and demands a theoretical explanation, so that we propose another scenario, which might overcome the aforementioned restrictions that forbid the appearance of electron-neutrinos to appear at low energy for the MiniBooNE experiments. We will present the scenario in our next work.

*Acknowledgements.* This work was supported in part by NNSFC under contract Nos. 10475042, 10745002 and 10705015 and the special foundation of the Education Ministry of China.

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